Harsanyi's aggregation theorem with incomplete preferences

Eric Danan, Thibault Gajdos, Jean-Marc Tallon

Documents de Travail du
Centre d'Economie de la Sorbonne

CES Working Papers

Harsanyi 's aggregation theorem with incomplete preferences

Eric DANAN, Thibault GAJDOS, Jean-Marc TALLON

2012.82

Maison des Sciences Économiques, 106-112 boulevard de L'Hôpital, 75647 Paris Cedex 13
http://centredeconomiesorbonne.univ-paris1.fr/bandeau-haut/documents-de-travail/
ISSN : 1955-611X

Harsanyi's aggregation theorem with incomplete preferences*

Eric Danan[†] Thibault Gajdos[‡] Jean-Marc Tallon[§]

December 3, 2012

Abstract

We provide a generalization of Harsanyi (1955)'s aggregation theorem to the case of incomplete preferences at the individual and social level. Individuals and society have possibly incomplete expected utility preferences that are represented by sets of expected utility functions. Under Pareto indifference, social preferences are represented through a set of aggregation rules that are utilitarian in a generalized sense. Strengthening Pareto indifference to Pareto preference provides a refinement of the representation.

Keywords. Incomplete preferences, aggregation, expected multi-utility, utilitarianism.
JEL Classification. D71, D81.

Introduction

Harsanyi (1955)'s aggregation theorem establishes that when individuals and society have expected utility preferences over lotteries, society's preferences can be represented by a weighted sum of individual utilities as soon as a Pareto indifference condition is satisfied. This celebrated result has become a cornerstone of social choice theory, being a positive aggregation result in a field where impossibility results are the rule, and is viewed by many as a strong argument in favor of utilitarianism.

Harsanyi's result sparked a rich (and on-going) debate about both its formal structure and substantive content (for an overview see, among others, Sen, 1986; Weymark, 1991;

*Financial support from ANR ComSoc (ANR-09-BLAN-0305-03) is gratefully acknowledged.

[†]THEMA, Université de Cergy-Pontoise, 33 boulevard du Port, 95011 Cergy-Pontoise cedex, France. E-mail: eric.danan@u-cergy.fr.

[‡]GREQAM, CNRS, Aix-Marseille University. E-mail: thibault.gajdos@univ-amu.fr.

[§]Paris School of Economics, Université Paris I Panthéon-Sorbonne, CNRS. E-mail: jean-marc.tallon@univ-paris1.fr.

Mongin and d'Aspremont, 1998; Fleurbaey and Mongin, 2012). An important question, in particular, is how robust the result is to more general preference specifications. Most findings on this issue are negative. For instance, Seidenfeld, Kadane, and Schervish (1989) and Mongin (1995) proved that moving from (objective) expected utility preferences over lotteries to subjective expected utility preferences over acts results in an impossibility unless all individuals share the same beliefs. Gajdos, Tallon, and Vergnaud (2008) showed that this impossibility extends even to the common belief case whenever individual preferences are not necessarily neutral towards ambiguity (as are subjective expected utility preferences).

In this note we take issue with the assumption of complete preferences. There are at least two reasons why one may want to allow for incomplete preferences in social choice theory. First, individuals may sometimes be intrinsically indecisive, i.e. unable to rank alternatives (Aumann, 1962; Bewley, 1986; Ok, 2002; Dubra, Maccheroni, and Ok, 2004; Ok, Ortoleva, and Riella, 2012). Second, even if individuals all have complete preferences, these preferences may in practice be only partially identified (Manski, 2005, 2011). As we shall see, Paretian aggregation remains possible when individual have incomplete expected utility preferences over lotteries, and still has a utilitarian flavor, although in a generalized sense.

Statement of the theorem

Let X be a finite set of *outcomes* and P denote the set of all probability distributions (*lotteries*) over X. A *utility function* on X is an element of $\mathbb{R}^X$. We denote by $e \in \mathbb{R}^X$ the constant utility function $x \mapsto e(x) = 1$.

Dubra, Maccheroni, and Ok (2004) show that a (weak) preference relation $\succsim$ over P satisfies the reflexivity, transitivity, independence, and continuity axioms if and only if it admits an *expected multi-utility representation*, i.e. a closed and convex set $U \subseteq \mathbb{R}^X$ such that for all $p, q \in P$,

$$p \succsim q \Leftrightarrow \left[\forall u \in U, \sum_{x \in X} p(x)u(x) \geq \sum_{x \in X} q(x)u(x) \right],$$

and that, moreover, $\mathrm{cl}(\mathrm{cone}(U) + \{\gamma e\}_{\gamma \in \mathbb{R}})$ is unique.[1] These are the standard axioms of the expected utility model (von Neumann and Morgenstern, 1944), except that completeness is weakened to reflexivity (and continuity is slightly strengthened). Thus, given these axioms, $\succsim$ is complete if and only if U can be taken to be a singleton, i.e. a standard *expected utility representation*, which is then unique up to positive affine transformations.

Call an expected multi-utility representation U *regular* if $\mathrm{cone}(U + \{\gamma e\}_{\gamma \in \mathbb{R}})$ is closed.

[1] $\mathrm{cl}(\cdot)$ denotes closure, $\mathrm{cone}(\cdot)$ denotes conical hull, and the sum of two sets is the Minkowski sum.

Any preference relation admitting an expected multi-utility representation also admits a regular one (and any expected utility representation is obviously regular), so we may without loss of generality restrict attention to such representations.

Consider a society made of a finite set $\{1, \dots, I\}$ of individuals. Each individual $i = 1, \dots, I$ is endowed with a (weak) preference relation $\succsim_i$ over P satisfying the above axioms. Society itself is also endowed with a preference relation $\succsim_0$ over P satisfying these axioms. For all $i = 0, \dots, I$, denote by $\succ_i$ and $\sim_i$ the asymmetric (strict preference) and symmetric (indifference) parts of $\succsim_i$, respectively. Say that the preference profile $(\succsim_i)_{i=0}^I$ satisfies *Pareto indifference* if for all $p, q \in P$, $[\forall i = 1, \dots, I, p \sim_i q] \Rightarrow p \sim_0 q$, and *Pareto preference* if for all $p, q \in P$, $[\forall i = 1, \dots, I, p \succsim_i q] \Rightarrow p \succsim_0 q$.

Harsanyi (1955)'s aggregation theorem establishes that if $\succsim_i$ is complete and endowed with an expected utility representation u_i for all $i = 0, \dots, I$, then (a) $(\succsim_i)_{i=0}^I$ satisfies Pareto indifference if and only if $u_0 = \sum_{i=1}^I \theta_i u_i + \gamma e$ for some $\theta \in \mathbb{R}^I$ and $\gamma \in \mathbb{R}$, (b) $(\succsim_i)_{i=0}^I$ satisfies Pareto preference if and only if the same holds with $\theta \in \mathbb{R}_+^I$.[2] Thus, in the expected utility setting, Pareto indifference (resp. preference) is necessary and sufficient for the social utility function to consist of a *signed utilitarian* (resp. *utilitarian*) aggregation of individual utility functions.

More generally, let us now endow $\succsim_i$ with an expected multi-utility representation U_i for all $i = 0, \dots, I$. This allows for preference incompleteness at both the individual and social level. We then obtain the following generalization of Harsanyi's aggregation theorem (whose proof is presented in the Appendix).

Theorem. Let $\succsim_i$ be a preference relation over P endowed with a regular expected multi-utility representation U_i, for all $1, \dots, I$.

(a) $(\succsim_i)_{i=0}^I$ satisfies Pareto indifference if and only if

$$U_0 = \left\{ \sum_{i=1}^I \alpha_i u_i - \beta_i v_i + \gamma e : \left(\alpha, \beta, \gamma, (u_i, v_i)_{i=1}^I\right) \in \Phi \right\} \tag{1}$$

for some closed set $\Phi \subseteq \mathbb{R}_+^{2I} \times \mathbb{R} \times \prod_{i=1}^I U_i^2$ with convex (α, β)-sections and convex $(u_i, v_i)_{i=1}^I$-sections.[3]

(b) If $\sum_{i=1}^I \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$ is closed, then $(\succsim_i)_{i=0}^I$ satisfies Pareto preference if and only if

$$U_0 = \left\{ \sum_{i=1}^I \theta_i u_i + \gamma e : \left(\theta, \gamma, (u_i)_{i=1}^I\right) \in \Omega \right\} \tag{2}$$

[2] See e.g. de Meyer and Mongin (1995) for a rigorous proof in a general setting.

[3] A set $S \subseteq S_1 \times S_2$ has convex s_1-sections if $\{s_2 \in S_2 : (s_1, s_2) \in S\}$ is convex for all s_1 in S_1.

for some closed set $\Omega \subseteq \mathbb{R}_+^I \times \mathbb{R} \times \prod_{i=1}^I U_i$ with convex θ-sections and convex $(u_i)_{i=1}^I$-sections.

Thus, in the expected multi-utility setting, Pareto indifference (resp. preference) is necessary and sufficient for the set of social utility functions to consist of a set of *bi-utilitarian* (resp. *utilitarian*) aggregations of individual utility functions. Bi-utilitarianism aggregates two utility functions u_i and v_i for each individual $i = 1, \ldots, I$, the former with a non-negative weight α_i and the latter with a non-positive weight $-\beta_i$, thereby generalizing signed utilitarianism (which corresponds to the particular case where $u_i = v_i$ for all $i = 1, \ldots, I$).[4]

Comments

Bi-utilitarianism cannot in general be reduced to signed utilitarianism in part (a) of the theorem, as the following example shows. Let $X = \{x, y, z, w\}$, $I = 2$, $U_0 = \{u_0\}$, $U_1 = \{u_1\}$, and $U_2 = \text{conv}(\{u_2^a, u_2^b\})$, where u_0, u_1, u_2^a, u_2^b are as follows.[5]

	u_0	u_1	u_2^a	u_2^b
x	4	1	1	-1
y	1	1	0	0
z	1	0	1	1
w	0	0	0	0

Then for all $p, q \in P$, $[\forall i = 1, 2, p \sim_i q] \Leftrightarrow p = q$, so $(\succsim_i)_{i=0}^2$ trivially satisfies Pareto indifference (consistently with the theorem, we have $u_0 = u_1 + 2u_2^a - u_2^b$). Yet there exists no $(\theta, \gamma, (u_i)_{i=1}^2) \in \mathbb{R}^2 \times \mathbb{R} \times \prod_{i=1}^2 U_i$ such that $u_0 = \sum_{i=1}^2 \theta_i u_i + \gamma e$.

The assumption that $\sum_{i=1}^I \text{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$ is closed in part (b) is not innocuous in terms of preference (unlike the regularity assumption on all U_i's), but there are at least two cases where it is automatically satisfied (details are provided in the appendix). The first is when $\succsim_i$ satisfies an additional *finiteness* axiom for all $i = 1 \ldots, I$ (Dubra and Ok, 2002). The second is when $(\succsim_i)_{i=1}^I$ satisfies a *minimal agreement* condition.

If $\succsim_i$ is complete and endowed with an expected utility representation u_i for all $i = 1, \ldots, I$, then (1) reduces to $U_0 = \{\sum_{i=1}^I \theta_i u_i + \gamma e : (\theta, \gamma) \in \Lambda\}$ for some closed and convex set $\Lambda \subseteq \mathbb{R}^I \times \mathbb{R}$, and (2) to the same with $\Lambda \subseteq \mathbb{R}_+^I \times \mathbb{R}$. On the other hand, if $\succsim_0$ is complete and endowed with an expected utility representation u_0, then (1) reduces to $u_0 = \sum_{i=1}^I \alpha_i u_i - \beta_i v_i + \gamma e$ for some $(\alpha, \beta, \gamma, (u_i, v_i)_{i=1}^I) \in \mathbb{R}_+^{2I} \times \mathbb{R} \times \prod_{i=1}^I U_i^2$, and (2) to $u_0 = \sum_{i=1}^I \theta_i u_i + \gamma e$ for some $(\theta, \gamma, (u_i)_{i=1}^I) \in \mathbb{R}_+^I \times \mathbb{R} \times \prod_{i=1}^I U_i$. Harsanyi's aggregation theorem is the intersection of these two particular cases.

[4]See Danan, Gajdos, and Tallon (2012) for a similar pattern in a multi-profile setting.
[5]conv$(\cdot)$ denotes convex hull.

These two cases have in common that $\Phi = \Psi \times W$ for some closed and convex sets $\Psi \subseteq \mathbb{R}^{2I}_+ \times \mathbb{R}$ and $W \subseteq \prod_{i=1}^I U_i^2$ in (1), and $\Omega = \Lambda \times V$ for some closed and convex sets $\Lambda \subseteq \mathbb{R}^I_+ \times \mathbb{R}$ and $V \subseteq \prod_{i=1}^I U_i$ in (2). Such a separation between weights and utilities is not always possible. This can be shown from the example above if we now let $U_0 = \mathrm{conv}(\{u_0^a, u_0^b\})$, where $u_0^a = \frac{3}{4}u_1 + \frac{1}{4}u_2^a$ and $u_0^b = \frac{1}{4}u_1 + \frac{3}{4}u_2^b$. Then $(\succsim_i)_{i=0}^I$ clearly satisfies Pareto preference, yet any Ω satisfying (2) contains both $((\frac{3}{4}, \frac{1}{4}), 0, (u_1, u_2^a))$ and $((\frac{1}{4}, \frac{3}{4}), 0, (u_1, u_2^b))$ but neither $((\frac{3}{4}, \frac{1}{4}), 0, (u_1, u_2^b))$ nor $((\frac{1}{4}, \frac{3}{4}), 0, (u_1, u_2^a))$.

Seeking a general characterization, in terms of the preference profile $(\succsim_i)_{i=0}^I$, of the possibility of separating weights and utilities in the above sense does not seem a promising avenue of research. Such a separation can be obtained in a multi-profile setting, by means of an additional *independence of irrelevant alternatives* condition linking distinct profiles $(U_i)_{i=0}^I$ with one another (Danan, Gajdos, and Tallon, 2012). This latter principle, however, also implies that $W = \prod_{i=1}^I U_i^2$ in (1) and $V = \prod_{i=1}^I U_i$ in (2). It is an open problem to find weaker conditions allowing society to make a selection within the individual sets of utility functions (thereby reducing social incompleteness) while retaining the separation between weights and utilities.

Appendix

Proof of the theorem

The "if" statements of both parts of the theorem are obvious, so we only prove the "only if" statements. To this end we first recall the following result from Dubra, Maccheroni, and Ok (2004).

Lemma. A preference relation $\succsim$ over P admits an expected multi-utility representation if and only if there exists a closed and convex cone $K \subseteq \mathbb{R}^X$, $K \perp \{\gamma e\}_{\gamma \in \mathbb{R}}$, such that for all $p, q \in P$, $p \succsim q \Leftrightarrow p - q \in K$.[6] Moreover, K is unique, and a closed and convex set $U \subseteq \mathbb{R}^X$ is an expected multi-utility representation of $\succsim$ if and only if $\mathrm{cl}(\mathrm{cone}(U) + \{\gamma e\}_{\gamma \in \mathbb{R}}) = K^*$.[7]

We start with part (b), so assume $\sum_{i=1}^I \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$ is closed and $(\succsim_i)_{i=0}^I$ satisfies Pareto preference. It is sufficient to show that for all $u_0 \in U_0$, there exist $\theta \in \mathbb{R}^I_+$, $\gamma \in \mathbb{R}$, and $u_i \in U_i$ for all $i = 1, \dots, I$ such that $u_0 = \sum_{i=1}^I \theta_i u_i + \gamma e$. Indeed, if this claim is correct then the set

$$\Omega = \left\{ (\theta, \gamma, (u_i)_{i=1}^I) \in \mathbb{R}^I_+ \times \mathbb{R} \times \prod_{i=1}^I U_i : \sum_{i=1}^I \theta_i u_i + \gamma e \in U_0 \right\}$$

[6] $\perp$ denotes orthogonality.

[7] K^* denotes the dual cone of K, i.e. $K^* = \{u \in \mathbb{R}^X : \forall k \in K, \sum_{x \in X} k(x)u(x) \geq 0\}$.

satisfies (2) by construction and is closed with convex θ-sections and convex $(u_i)_{i=1}^I$-sections since U_0 is closed and convex.

To prove the claim, let K_i be the closed and convex cone corresponding to $\succsim_i$ in the lemma above, for all $i = 0, \ldots, I$. We then have $\cap_{i=1}^I K_i \subseteq K_0$ by Pareto preference and, hence, $K_0^* \subseteq (\cap_{i=1}^I K_i)^* = \mathrm{cl}(\sum_{i=1}^I K_i^*)$ (Rockafellar, 1970, Corollary 16.4.2). Moreover, for all $i = 0, \ldots, I$, since U_i is regular we also have $K_i^* = \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$ by the lemma above. Hence

$$\begin{aligned} U_0 \subseteq \mathrm{cone}(U_0) + \{\gamma e\}_{\gamma\in\mathbb{R}} &\subseteq \mathrm{cl}\left(\sum_{i=1}^I (\mathrm{cone}(U_i) + \{\gamma e\}_{\gamma\in\mathbb{R}})\right) \\ &= \mathrm{cl}\left(\sum_{i=1}^I \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma\in\mathbb{R}}\right) \\ &= \sum_{i=1}^I \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma\in\mathbb{R}}, \end{aligned}$$

where the last equality follows from the assumption that $\sum_{i=1}^I \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma\in\mathbb{R}}$ is closed. Hence for all $u_0 \in U_0$, there exist $\gamma \in \mathbb{R}$ and $u_i' \in \mathrm{cone}(U_i)$ for all $i = 1, \ldots, I$ such that $u_0 = \sum_{i=1}^I u_i' + \gamma e$. Moreover, for all $i = 1, \ldots, I$, since U_i is convex we also have $u_i' = \theta_i u_i$ for some $\theta_i \in \mathbb{R}_+$ and $u_i \in U_i$ and, hence, $u_0 = \sum_{i=1}^I \theta_i u_i + \gamma e$.

Now for part (a), assume $(\succsim_i)_{i=0}^I$ satisfies Pareto indifference. As in part (b) it is sufficient to show that for all $u_0 \in U_0$, there exist $\alpha, \beta \in \mathbb{R}_+^I$, $\gamma \in \mathbb{R}$, and $u_i, v_i \in U_i$ for all $i = 1, \ldots, I$ such that $u_0 = \sum_{i=1}^I \alpha_i u_i - \mu_i v_i + \gamma e$. To prove this, define the preference relation $\succsim_i'$ over P by $p \succsim_i' q \Leftrightarrow p \sim_i q$, for all $i = 1, \ldots, I$. We then have $p \succsim_i' q \Leftrightarrow p - q \in K_i \cap (-K_i)$, and $(\succsim_0, (\succsim_i)_{i=1}^I)$ obviously satisfies Pareto preference, so by the same argument as in the proof of part (b) we obtain $K_0^* \subseteq \mathrm{cl}(\sum_{i=1}^I (K_i \cap (-K_i))^*) = \mathrm{cl}(\sum_{i=1}^I (K_i^* - K_i^*))$. Hence

$$\begin{aligned} U_0 \subseteq \mathrm{cone}(U_0) + \{\gamma e\}_{\gamma\in\mathbb{R}} &\subseteq \mathrm{cl}\left(\sum_{i=1}^I (\mathrm{cone}(U_i) + \{\gamma e\}_{\gamma\in\mathbb{R}} - \mathrm{cone}(U_i) - \{\gamma e\}_{\gamma\in\mathbb{R}})\right) \\ &= \mathrm{cl}\left(\sum_{i=1}^I \mathrm{cone}(U_i) - \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma\in\mathbb{R}}\right) \\ &= \sum_{i=1}^I \mathrm{cone}(U_i) - \mathrm{cone}(U_i) + \{\gamma e\}_{\gamma\in\mathbb{R}}, \end{aligned}$$

where the last equality follows from the fact that $\mathrm{cone}(U_i) - \mathrm{cone}(U_i)$ is a subspace of $\mathbb{R}^X$ and, hence, is closed. Hence for all $u_0 \in U_0$, there exist $\gamma \in \mathbb{R}$ and $u_i', v_i' \in \mathrm{cone}(U_i)$ for all $i = 1, \ldots, I$ such that $u_0 = \sum_{i=1}^I u_i' - v_i' + \gamma e$. Moreover, for all $i = 1, \ldots, I$, since U_i is convex we also have $u_i' = \alpha_i u_i$ and $v_i' = \beta_i v_i$ for some $\alpha_i, \beta_i \in \mathbb{R}_+$ and $u_i, v_i \in U_i$ and, hence, $u_0 = \sum_{i=1}^I \alpha_i u_i - \beta_i v_i + \gamma e$.

On the closedness assumption in part (b) of the theorem

As can be seen from the proof of part (b), the closedness assumption ensures that each social utility function can be expressed as a non-negative linear combination of some individual utility functions (and a constant function). Indeed, even though each $\text{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$ is closed, their sum is not necessarily closed in general. There are at least two cases, however, where the sum is guaranteed to be closed.

The first case is when each $\text{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$ is polyhedral (Rockafellar, 1970, Corrolary 19.3.2). This is equivalent to the corresponding K_i in the lemma above being polyhedral (Rockafellar, 1970, Corrolary 19.2.2), and can be characterized by a *finiteness* axiom on $\succsim_i$ (Dubra and Ok, 2002). Note that no closedness assumption is needed in part (a) because $\text{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$ is replaced with $\text{cone}(U_i) - \text{cone}(U_i) + \{\gamma e\}_{\gamma \in \mathbb{R}}$, which is a subspace of $\mathbb{R}^X$ and, hence, falls into this case.

The second case is when all K_i's have a common point in their relative interiors (Rockafellar, 1970, Corrolary 16.4.2). This can be characterized by the following *minimal agreement* condition: there exist $p, q \in P$ such that $p \succsim_i^* q$ for all $i \in I$, where $p \succsim_i^* q$ is defined by for all $q_i \in P$ such that $p \succsim_i q_i$, there exist $q_i' \in P$ and $\lambda_i \in (0,1)$ such that $p \succsim_i q_i'$ and $q = \lambda_i q_i + (1-\lambda_i) q_i'$. Note that if all $\succsim_i$'s are complete then this condition boils down to the usual minimal agreement condition, where $p \succsim_i^* q$ is replaced with $p \succ_i q$.

References

Aumann, R. (1962): "Utility theory without the completeness axiom," *Econometrica*, 30(3), 445–462.

Bewley, T. (1986): "Knightian decision theory: Part I," Discussion Paper 807, Cowles Foundation Discussion Papers, published in Decisions in Economics and Finance (2002), 25, 79-110.

Danan, E., T. Gajdos, and J.-M. Tallon (2012): "Aggregating sets of von Neumann-Morgenstern utilities," *Journal of Economic Theory*, forthcoming.

de Meyer, B., and P. Mongin (1995): "A note on affine aggregation," *Economics Letters*, 47, 177–183.

Dubra, J., F. Maccheroni, and E. A. Ok (2004): "Expected utility theory without the completeness axiom," *Journal of Economic Theory*, 115(1), 118–133.

Dubra, J., and E. Ok (2002): "A model of procedural decision making in the presence of risk," *International Economic Review*, 43(4), 1053–1080.

Fleurbaey, M., and P. Mongin (2012): "The utilitarian relevance of the aggregation theorem," mimeo.

Gajdos, T., J.-M. Tallon, and J.-C. Vergnaud (2008): "Representation and aggregation of preferences under uncertainty," *Journal of Economic Theory*, 141(1), 68–99.

Harsanyi, J. C. (1955): "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility," *Journal of Political Economy*, 63(4), 309–321.

Manski, C. (2005): *Social choice with partial knowledge of treatment responses*. Princeton University Press.

——— (2011): "Policy choice with partial knowledge of policy effectivemess," *Journal of Experimental Criminology*, 7(2), 111–125.

Mongin, P. (1995): "Consistent Bayesian aggregation," *Journal of Economic Theory*, 66(2), 313–351.

Mongin, P., and C. d'Aspremont (1998): "Utility theory and ethics," in *Handbook of Utility Theory, I*, ed. by S. Barberà, P. Hammond, and C. Seidl, pp. 233–289. Kluwer, Dordrecht.

Ok, E. A. (2002): "Utility representation of an incomplete preference relation," *Journal of Economic Theory*, 104(2), 429–449.

Ok, E. A., P. Ortoleva, and G. Riella (2012): "Incomplete preferences under uncertainty: Indecisiveness in beliefs versus tastes," *Econometrica*, 80(4), 1791–1808.

Rockafellar, R. T. (1970): *Convex analysis*. Princeton University Press.

Seidenfeld, T., J. B. Kadane, and M. J. Schervish (1989): "On the shared preferences of two Bayesian decision makers," *Journal of Philosophy*, 86(5), 225–244.

Sen, A. K. (1986): "Social choice theory," in *Handbook of Mathematical Economics*, ed. by K. J. Arrow, and M. D. Intriligator, vol. III, chap. 22, pp. 1073–1191. North-Holland.

von Neumann, J., and O. Morgenstern (1944): *Theory of games and economic behavior*. Princeton University Press.

Weymark, J. A. (1991): "A reconsideration of the Harsanyi-Sen debate on utilitarianism," in *Interpersonnal Comparisons of Well-Being*, ed. by J. Elster, and J. E. Roemer, pp. 255–320, Cambridge. Cambridge University Press.